TKU-HEP 95/02
OCHA-PP-62
February 2008

Constraints on Light Top Squark from B^0-$\bar{B}^0$ mixing

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Abstract

We discuss the constraints on the mass of the lighter top squark from B^0-$\bar{B}^0$ mixing in the minimal supersymmetric standard model. A light top squark whose mass is less than half of the Z^0-boson mass has not yet been excluded from direct search experiments at LEP. However, the existence of the light top squark may exceedingly enhance B^0-$\bar{B}^0$ mixing, owing to the box diagrams exchanging the charginos and the up-type squarks. We show that for a sizable region of parameter space the light top squark contribution to B^0-$\bar{B}^0$ mixing becomes the same order of magnitude as the standard W-boson contribution. Taking into account the experimental results for B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings, the existence of the light top squark is excluded in an appreciable region of the parameter space which LEP experiments have not ruled out.

*Research Fellow of the Japan Society for the Promotion of Science.

†Supported by Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture, contract number 6640409.

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Supersymmetric particles have been searched for extensively in experiments, since supersymmetry (SUSY) is considered one of the most promising ideas for physics beyond the standard model (SM) from both aesthetical and phenomenological viewpoints. Up to now no direct experimental evidence for those new particles has been found, giving constraints on SUSY parameters [1]. In particular, abundant data of e^+e^- collisions at LEP almost exclude pair productions of charged SUSY particles at the Z^0 resonance. However, there still remains a possibility that one of the top squarks (hereafter, we say “stop” for short) has a mass less than the half of the Z^0-boson mass.

In the minimal supersymmetric standard model (MSSM) one stop could naturally be lighter than the other squarks [2]. Because, the large mass of the top quark induces a large mixing between the left- and the right-handed stops, while such a mixing is negligible in the first and second generations. One of the stops could be the lightest charged SUSY particle. Moreover, if the left-right mixing angle of the stops has a certain value, the lighter stop decouples with the Z^0-boson [3]. Then the cross section of the stop pair production is not large at the Z^0 resonance, and the stop could escape detection at LEP.

The experimental mass bounds on the lighter stop $\tilde{t}_1$ have been given by CELLO at PETRA [4], AMY, TOPAZ, VENUS at TRISTAN [5], and OPAL at LEP [6], which depend on the assumed mass difference Δm between the stop and the lightest neutralino. Assuming $\Delta m \geq 5$ GeV, the lower bound is about 45 GeV, if the left-right mixing angle has a value not close to the ‘decoupling’ value. On the other hand, if this mixing angle lies in the vicinity of the ‘decoupling’ value, the lower bound is about 40 GeV for $\Delta m \geq 5$ GeV and 25 GeV for $\Delta m \geq 2$ GeV. There also have appeared several analyses which constrain the light stop mass from other phenomena, such as the decay $b \to s\gamma$ [7] and the cosmological mass density [8]. However, the allowed region derived from the e^+e^- experiments is not much altered.

In this letter we discuss the possibility of the existence of the light stop, whose mass is smaller than $\frac{1}{2}M_Z$, by investigating its effects on B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings. In the MSSM, these mixings receive contributions from the box diagrams mediated by the charginos and up-type squarks and those mediated by the charged Higgs bosons and up-type quarks, as well as the standard box diagrams. It was recently shown [9] that these new contributions are comparable to the SM contributions in sizable regions of the SUSY parameter space. In particular, the chargino

contributions generally become large if one stop is much lighter than the other up-type squarks, owing to less efficient cancellation among different squark contributions. On the other hand, such theoretical predictions for B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings can be well examined by experiments available at present or in the near future [10], which may constrain or reveal the new contributions by the MSSM. The aim of this paper is to study the new contributions, concentrating on the case where a light stop exists and decouples with the Z^0-boson. We will show, as a conclusion, that within the SUSY parameter space consistent with the light stop, there are wide regions which are not consistent with the present experimental data for B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings or will be explored by future experiments at B-factories.

First, we briefly review the masses and mixings of the squarks [11], assuming $N = 1$ supergravity and grand unification [12]. At the electroweak scale the interaction eigenstate squarks are mixed in generation space through their mass terms. For the up-type squarks these generation mixings in the mass-squared matrix are approximately removed by the same matrices that diagonalize the mass matrix of the up-type quarks. Consequently, the 'super' Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the generation mixings in the interactions of the down-type quark, up-type squark, and chargino in mass eigenstate basis, is the same as the CKM matrix of the quarks.

In each flavor the left-handed and right-handed squarks are mixed by the Yukawa interaction, which is proportional to the corresponding quark mass. From the smallness of the u- and c-quark masses, these mixings can be safely neglected for the first two generations. The masses of the left-handed squarks $\tilde{u}_L, \tilde{c}_L$ and the right-handed squarks $\tilde{u}_R, \tilde{c}_R$ are given by

$$\begin{aligned} m_{\tilde{u}L}^2 &= m_{\tilde{c}L}^2 = \tilde{m}_Q^2 + \cos 2\beta(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)M_Z^2, \\ m_{\tilde{u}R}^2 &= m_{\tilde{c}R}^2 = \tilde{m}_U^2 + \frac{2}{3}\cos 2\beta \sin^2\theta_W M_Z^2, \\ & \tan\beta = \frac{v_2}{v_1}, \end{aligned} \tag{1}$$

where v_1 and v_2 stand for the vacuum expectation values of the Higgs bosons with the hypercharges $-\frac{1}{2}$ and $\frac{1}{2}$, respectively. The mass parameters $\tilde{m}_Q$ and $\tilde{m}_U$ are determined by the gravitino mass $m_{3/2}$ and the gaugino masses, and $\tilde{m}_Q \simeq \tilde{m}_U \sim m_{3/2}$. For the third generation, an appreciable mixing between $\tilde{t}_L$ and $\tilde{t}_R$ is induced by the large top quark mass m_t. The mass-squared matrix for the stops is given

by

$$M_{\tilde{t}}^2 = \left(\begin{array}{cc} m_{\tilde{u}L}^2 + (1-|c|)m_t^2 & (\cot\beta m_H + a^* m_{3/2})m_t \\ (\cot\beta m_H^* + a m_{3/2})m_t & m_{\tilde{u}R}^2 + (1-2|c|)m_t^2 \end{array} \right), \tag{2}$$

where m_H denotes the higgsino mass parameter. If the SU(2)$\times$U(1) symmetry is broken through radiative corrections, the magnitude of m_H is at most of order of $m_{3/2}$, and $\tan\beta \gtrsim 1$. The dimensionless constants a and c depend on other SUSY parameters: a is related to the breaking of local supersymmetry and its absolute value is of order of unity; c is related to radiative corrections to the squark masses and $|c| = 0.1 - 1$. The mass eigenstates of the stops $\tilde{t}_1$, $\tilde{t}_2$ are obtained by diagonalizing the matrix $M_{\tilde{t}}^2$ as

$$S_t M_{\tilde{t}}^2 S_t^\dagger \;=\; \mathrm{diag}(m_{\tilde{t}1}^2, m_{\tilde{t}2}^2) \;\; (m_{\tilde{t}1}^2 < m_{\tilde{t}2}^2). \tag{3}$$

The unitary matrix S_t and the stop masses $m_{\tilde{t}i}$ are given by

$$\begin{aligned} S_t &= \left(\begin{array}{cc} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{array} \right), \\ \tan\theta_t &= \frac{2(M_{\tilde{t}}^2)_{12}}{-(M_{\tilde{t}}^2)_{11} + (M_{\tilde{t}}^2)_{22} + \sqrt{D}}, \\ m_{\tilde{t}1(2)}^2 &= \frac{1}{2}\{(M_{\tilde{t}}^2)_{11} + (M_{\tilde{t}}^2)_{22} - (+)\sqrt{D}\}, \\ & \quad D = \{(M_{\tilde{t}}^2)_{11} - (M_{\tilde{t}}^2)_{22}\}^2 + 4\{(M_{\tilde{t}}^2)_{12}\}^2, \end{aligned} \tag{4}$$

where SUSY parameters are assumed to have real values.

Let us consider the conditions which SUSY parameters should satisfy in order for a stop to have a small mass and decouple with the Z^0-boson. The interaction Lagrangian for the lighter stop and the Z^0-boson is given by

$$\mathcal{L} = -i\frac{e}{\sin 2\theta_W}(\cos^2\theta_t - \frac{4}{3}\sin^2\theta_W) Z_\mu \tilde{t}_1^* \partial^\mu \tilde{t}_1 + \mathrm{h.c.}, \tag{5}$$

which vanishes for $\cos^2\theta_t = \frac{4}{3}\sin^2\theta_W$. If the mixing angle θ_t has this 'decoupling' value, SUSY parameters satisfy the following equations:

$$\begin{aligned} \tilde{m}_Q^2 &= \left(-1 + \frac{2-3r}{1-2r}|c|\right) m_t^2 + m_{\tilde{t}1}^2, && (6) \\ m_H &= \tan\beta\{-a m_{3/2} \pm \sqrt{r(1-r)}\left(\frac{1}{2}\cos 2\beta \frac{M_Z^2}{m_t^2} + \frac{|c|}{1-2r}\right) m_t\}, && (7) \end{aligned}$$

where $r = \frac{4}{3}\sin^2\theta_W$ and we have put $\tilde{m}_Q^2 = \tilde{m}_U^2$. The value of $\tilde{m}_Q^2$ is determined by $|c|$ and $m_{\tilde{t}1}$. The experimental lower bound on the squark masses of the first two

generations is about 150 GeV [13], which imposes the constraint $\tilde{m}_Q(\simeq \tilde{m}_U) > 150$ GeV, as seen from eq. (1). For $m_{\tilde{t}1} < 45$ GeV, this constraint leads to $|c| > 0.6$, whereas $|c|$ is theoretically at most unity, corresponding to $\tilde{m}_Q \simeq 230$ GeV. The squarks of the first two generations are thus predicted to have masses not much larger than 200 GeV. The value of m_H depends on $\tan\beta$ and $am_{3/2}$ as well as $|c|$. However, since the value of $am_{3/2}$ should not be much different from $\tilde{m}_Q$, only $\tan\beta$ is an independent parameter. In the MSSM, the lightest neutralino χ_1 has to be lighter than the light stop, which gives another condition for $\tan\beta$, m_H, and the SU(2) gaugino mass $\tilde{m}_2$. Therefore, if there exists a light stop which decouples with the Z^0-boson, the values of other SUSY parameters are restricted within narrow ranges.

The interactions of the chargino, up-type squark, and down-type quark and those of the charged Higgs boson, up-type quark, and down-type quark can add sizable new contributions to B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings through box diagrams. These short distance contributions are experimentally measured by the mixing parameter x_d for B^0-$\bar{B}^0$ mixing and the CP violation parameter ϵ for K^0-$\bar{K}^0$ mixing. Neglecting small new contributions, these parameters are theoretically given by

$$\begin{aligned} x_d &= \frac{G_F^2}{6\pi^2} M_W^2 \frac{M_B}{\Gamma_B} f_B^2 B_B |V_{31}^* V_{33}|^2 \eta_B |A_{tt}^W + A^C + A_{tt}^H|, &(8)\\ \epsilon &= -e^{i\pi/4} \frac{G_F^2}{12\sqrt{2}\pi^2} M_W^2 \frac{M_K}{\Delta M_K} f_K^2 B_K \mathrm{Im}[(V_{31}^* V_{32})^2 \eta_{K33} (A_{tt}^W + A^C + A_{tt}^H) \\ &\quad + (V_{21}^* V_{22})^2 \eta_{K22} A_{cc}^W + 2 V_{31}^* V_{32} V_{21}^* V_{22} \eta_{K32} A_{tc}^W], &(9) \end{aligned}$$

where f_B, B_B, and η_B represent the decay constant, the bag factor, and the QCD correction factor, respectively, for the B-meson, and f_K, B_K, and η_{Kab} represent those for the K-meson. The CKM matrix is denoted by V. The contributions of the W-boson, chargino, and charged Higgs boson box diagrams are respectively expressed as A^W's, A^C, and A_{tt}^H, which are explicitly given in refs. [10, 14]. In the SM, x_d and ϵ are given by eqs. (8) and (9) with $A^C = A_{tt}^H = 0$. The term proportional to $(V_{31}^* V_{32})^2$ in eq. (9) is enhanced by the same amount as x_d. The difference between the contributions of the MSSM and the SM can thus be measured by the ratio

$$R = \frac{A_{tt}^W + A^C + A_{tt}^H}{A_{tt}^W}. \qquad (10)$$

If new contributions are negligible, R becomes unity.

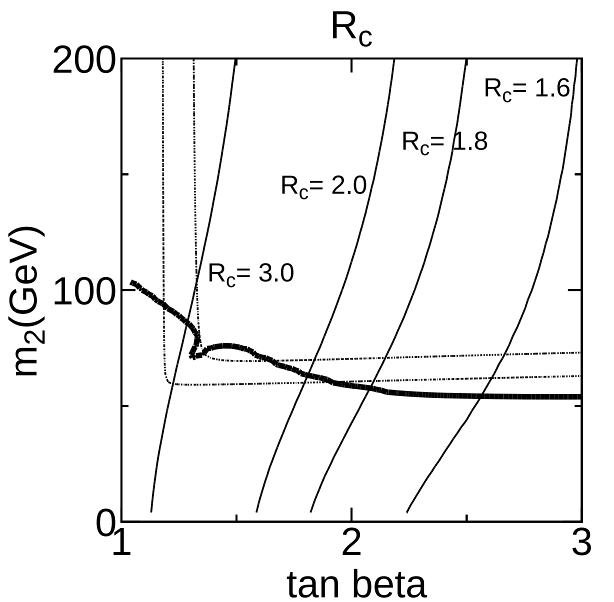

Figure 1: The contours of the ratio R_C for $m_{\tilde{t}1} = 40$ GeV and $\cos^2\theta_t = \frac{4}{3}\sin^2\theta_W$.

Now, we examine the effects of the light stop on B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings. In order to see the chargino contribution exclusively, we first consider the ratio $R_C = (A^W_{tt} + A^C)/A^W_{tt}$ instead of R. In Fig. 1 we plot contours of R_C in the $(\tan\beta, \tilde{m}_2)$ plane, taking $\cos^2\theta_t = \frac{4}{3}\sin^2\theta_W$ and $m_{\tilde{t}1} = 40$ GeV. The values of the other parameters are set for $|c| = 0.7$ and $\tilde{m}_Q = \tilde{m}_U = am_{3/2}$. For the t-quark mass we use $m_t = 170$ GeV [15]. When the input parameters are fixed, two values are possible for m_H from eq. (7). For $\tan\beta = 2$, the value of m_H becomes about -70 GeV or -620 GeV, which varies roughly in proportion to $\tan\beta$. We have taken the value of smaller magnitude. The magnitude of the other one would be too large, compared to $m_{3/2}$, for radiative breaking of the SU(2)$\times$U(1) symmetry. The region below the bold line is ruled out by the experimental bounds on the chargino mass $\tilde{m}_{\omega 1} > 45$ GeV and the Z^0-boson decay widths to the neutralinos $\sum_{i,j\neq 1}\Gamma(Z^0 \to \chi_1\chi_i, \chi_i\chi_j) < 5\times 10^{-2}$ MeV, $\Gamma(Z^0 \to \chi_1\chi_1) < 8.4$ MeV [16]. In the region below the lower dotted curve, the mass difference between the lightest neutralino and the lighter stop becomes larger than 5 GeV, which is ruled out by the recent direct stop searches [6]. In the region above the upper dotted curve, the lighter stop becomes lighter than the lightest neutralino, which is cosmologically disfavored.

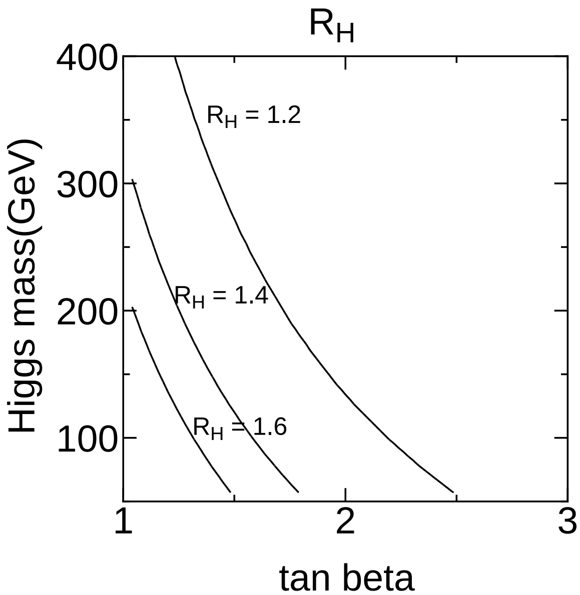

Figure 2: The contours of the ratio R_H.

Within the presently allowed region, i.e. the region between the two dotted curves above the bold line, a large value of R_C is predicted if $\tan\beta$ is not much larger than unity: $R_C \gtrsim 2$ for $\tan\beta \lesssim 2$ and $R_C \gtrsim 1.5$ for $\tan\beta \lesssim 3$. The chargino contribution and the W-boson contribution interfere constructively, so that $R_C > 1$. If there exists a light stop, the new contribution can be comparable to or even larger than the standard model contribution. The ratio R_C increases as $\tan\beta$ decreases, since a smaller value for v_2 enhances the Yukawa couplings of the charginos to the stops.

The net effect of the MSSM is given by summing all the contributions, for which the charged Higgs boson contribution has to be evaluated. In Fig. 2 we show the ratio $R_H = (A_{tt}^W + A_{tt}^H)/A_{tt}^W$ as contours in the $(\tan\beta, M_{H\pm})$ plane, $M_{H\pm}$ being the charged Higgs boson mass. The charged Higgs boson contribution also interferes constructively with the W-boson contribution. Therefore, the value of R in eq. (10) becomes larger than that of R_C shown in Fig. 1.

The value of R affects the evaluation of the CKM matrix elements through x_d and ϵ [10]. Adopting the standard parametrization [1], the independent parameters of the CKM matrix are represented by three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and one CP-violating phase δ. Experimentally, three mixing angles are determined by the

processes for which new contributions by the MSSM are negligible. On the other hand, the CP-violating phase is determined by x_d or ϵ, which depends on the value of R. Therefore, the value of δ in the MSSM with $R > 1$ is different from that in the SM of $R = 1$. Furthermore, x_d and ϵ have to consistently give a value of δ for a given R, which constrains the values allowed for δ and R.

The ranges of R and $\cos\delta$ which are consistent with the experimental results for x_d and ϵ have been discussed in ref. [10]. Taking into account theoretical uncertainties for hadronic matrix elements, it was shown that the allowed range is $1 \lesssim R \lesssim 2$ for the experimental central values of $\sin\theta_{13}$ and $\sin\theta_{23}$. Even if experimental uncertainties for these mixing angles are taken into account, the value of R is at most 3. It was also shown that the measurements of CP asymmetries of the B-meson decays at B-factories could further constrain the R value.

From these constraints on the value of R, we can see that most of the allowed region for $\tan\beta \lesssim 2$ in Fig. 1 is inconsistent with B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings. Hence, the existence of a light stop with $m_{\tilde{t}1} < M_Z/2$ can be ruled out, if $\tan\beta$ is around 2 or less. In the allowed region for $\tan\beta > 2$, the value of R becomes smaller than 2, which is not excluded from the present data for B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings. However, the allowed region for $\tan\beta \lesssim 3$, where $R > 1.5$, would be explored at B-factories in the near future.

In conclusion, we have discussed the effects of a light stop whose mass is less than $M_Z/2$ on B^0-$\bar{B}^0$ and K^0-$\bar{K}^0$ mixings. The existence of such a light stop gives large new contributions to those mixings in sizable regions of the MSSM parameter space. Examining these effects in the light of experimental measurements of the x_d and ϵ parameters, we have shown that the light stop can be ruled out if $\tan\beta \lesssim 2$. For $\tan\beta > 2$, there are still rooms for a light stop. However, further constraints will be obtained at B-factories.

We thank S. Komamiya for correspondence.

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